Table 3 Performance characteristics of a gaseous nuclear rocket[a]

	$\zeta_1 = 0.1$	$\zeta_1 = 0.3$	$\zeta_1 = 0.5$
Electrical power for MHD vortex, kw	3.309	17.4	87.9
Radial electric current, amp	66.2	348	1760
Average rotational velocity, m/sec	55.5	71.4	99.9
Average density ratio $\langle n_H/n_U \rangle$	54.8	107.6	147.1
Density ratio at inner wall (n_H/n_U)	2.2×10^3	1.844×10^4	2.042×10^4
Maximum permissible R_N	−60	−116	−202
Reynolds number used	−55	−115	−200
Mass flow rate, lb/sec	2.61	5.63	12.5
Thrust, lb	7830	16,890	37,500

[a] Specific impulse = 3000 sec; r_0 = 1 m; temperature = 1 ev; $n_{0U}n_{0H} = 1.5$; ϕ_0 = 50 v; B_0 = 1 weber/m²; mass of U^{235} = 15 kg.

For an axial magnetic field strength of 1 weber/m² and radial voltage difference of 50 v maintained between the concentric cylinders, we find the results for maximum permissible mass flow rate shown in Fig. 1. These results unfortunately correspond to very low propellant flow rates and constitute a major limitation to the thrust obtainable with a single vortex system. Table 3 summarizes the rocket characteristics.

The major limitation on this system appears to be the low value of radial mass flow resulting in low total thrust for a single vortex system. Even for moderate electrical power requirement, the theoretical containment of the uranium fuel is quite good: the concentration of U^{235} in the rocket exhaust is of the order of one-ten-thousandth of the concentration of monatomic hydrogen.

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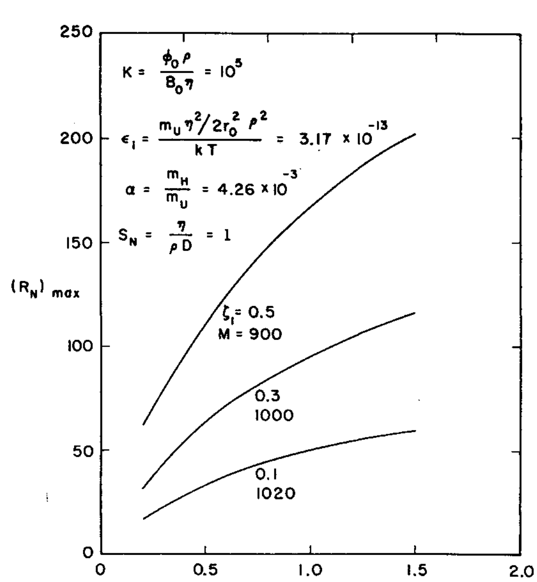

Fig. 1 Variation of the maximum permissible radial Reynolds number with ζ_1 vs boundary composition ratio.

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A Transformation for Wake Analyses

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Laminar near wakes are receiving much emphasis at present. It is the purpose of this note to show that analyses that already have been made for turbulent near wakes are of value in studying laminar near wakes by means of a simple transformation.

The equations for both turbulent and laminar jet mixing have been developed.[1] From those results, it can be observed that half-infinite two-dimensional laminar jet mixing is governed by equations that are identical to the turbulent case, if the turbulent similarity parameter σ is replaced by an equivalent laminar parameter and if the laminar viscosity is assumed constant. The transformation condition (turbulent-to-laminar or vice versa) which must be satisfied is

$$\sigma = [Ux/\nu\alpha]^{1/2}/2 \qquad (1)$$

where U is the adjacent potential velocity, x is the mixing region length, ν is the viscosity, and α is a reference perturbation velocity factor $(1.0 \leq \alpha \leq 2.0)$.

With the aid of this transformation, the tested and proved turbulent theories and solutions (e.g., Ref. 2) for situations in which the jet mixing is controlling (e.g., the base flow or near-wake problem) may be transferred to laminar constant viscosity conditions. Reference 3 applies as well for the laminar constant viscosity case as the turbulent case when the transformation law is used. If the viscosity cannot be considered constant in the mixing region, then results obtained by using this transformation must be considered as a first approximation.

As an example of application of the transformation, Fig. 1 shows the dimensionless jet-boundary-streamline velocity vs

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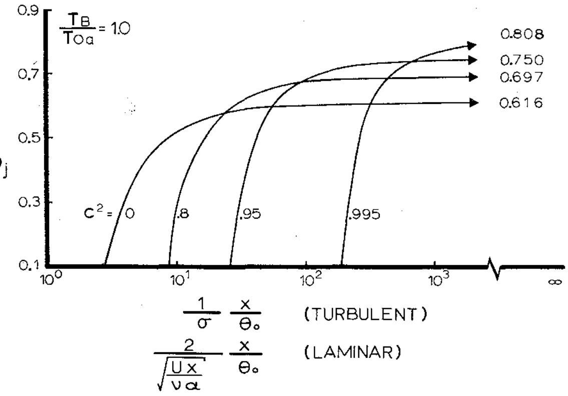

Fig. 1 Dimensionless velocity of dividing streamline for half-infinite isoenergetic jet mixing of a perfect gas.

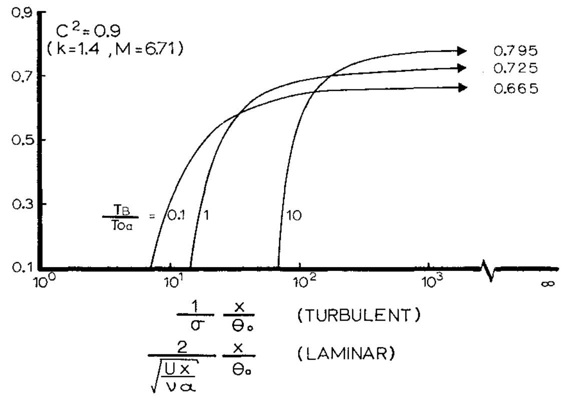

Fig. 2 Dimensionless velocity of dividing streamline for half-infinite diabatic jet mixing of a perfect gas.

either a laminar or turbulent abscissa for an adiabatic two-dimensional mixing region with the square of the Crocco number (U/U_{max}) of the adjacent flow as a parameter. (The equivalent mass bleed concept[4,5] was used for the calculations.) Regardless of the character of the flow, the abscissa is a measure of the length of the mixing region x in terms of the momentum thickness at the start of the mixing region θ_0. The limiting values of φ_j for fully developed mixing are also shown. As shown, smaller values of φ_j are obtained for developing mixing regions (i.e., those with initial boundary layers of finite thickness as compared with wake length).

Figure 2 shows an example with heat transfer for a perfect gas with Prandtl number of unity. It shows that the diabatic condition influences φ_j for the laminar and turbulent cases. The ratio of bulk temperature of the stagnant gas T_B to the total temperature of the adjacent flow T_{0a} is an important parameter.

In order to obtain numerical results from Figs. 1 and 2, α must be known. The integral method from which these two figures were obtained uses an error function solution of a simplified equation of motion to approximate the velocity profile appearing in the integrals. The simplified equation of motion may be obtained by a perturbation procedure from the boundary-layer equation of motion. If the procedure uses perturbations about the potential velocity $\alpha = 1.0$, and if about the average mixing region velocity, $\alpha = 2.0$. Although there are some indications[6] that α tends toward 2.0, further evidence is needed in order to determine α precisely.

Even without precise knowledge of α, Figs. 1 and 2 indicate an example of the usefulness of this turbulent-to-laminar transformation for adiabatic and diabatic mixing regions.

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Hypersonic Cone Wake Velocities Obtained from Streak Pictures

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THE deceleration of the flow in the turbulent wake of hypersonic conical bodies has been investigated and reported by several authors.[1,2] Slattery and Clay[1] obtained experimentally average velocities in cone wakes by following the histories of identifiable packages of gas at the edge of the turbulent front in high-speed schlieren moving pictures. Their data were resolvable starting at approximately 50 body diameters behind the cone and extended to some 3000 diameters downstream. Hromas and Lees[2] and Lien, Erdos, and Pallone[4] have calculated centerline velocities in cone wakes. These solutions cover the distance from the body to several hundreds of body diameters downstream.

For cone wakes, the region of maximum interest is near to the body because the disturbances in velocity and enthalpy produced by the cone are small, and these properties decay rapidly toward their freestream values under the influence of turbulent diffusion. For hypersonic conical flow fields, the maximum decay region for these properties is well within 25 body diameters for most cases of interest. By the streak drum camera technique,[5] it is possible to obtain data on representative cone wake velocities in the turbulent core in the important region between 5 and 25 diameters behind the body.

The cone wake velocity data in this paper were obtained from drum-camera streak photographs taken during experiments performed on Canadian Armament Research and Development Establishment (CARDE) Range 2. The projectiles were sharp 15° semivertex angle (θ_c) plastic (zelux) cones with base diameter equal to $\frac{1}{2}$ in. (One set of data was also obtained for a 45° semivertex angle cone.) Ranges of freestream properties from 0.88 to 7.6 cm Hg in pressure and from 14,000 to 16,000 fps in velocity were covered. A typical cone streak picture is shown in Fig. 1. The bright band was produced by the cone itself. The streaks, from which the wake velocity was measured, represent a flow unsteadiness.[7,8] These disturbances are made visible by the self-luminosity of

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